Satellite-to-Satellite Tracking Orbit Determination

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Error analyses and parametric studies of satellite-to-satellite tracking orbit determination in the 1980's are presented. Uncertainties in dynamic model parameters are found to be dominant error sources. The tracking satellite ephemeris errors are of course reflected as tracked satellite orbit errors, but the magnitudes are scaled down by a ratio of the orbital radii. These ephemeris errors, together with other systematic tracking errors, are secondary in importance, while random measurement noises are insignificant. As a consequence, tracking rates much reduced from those customarily used at present are permissible; orbit determination accuracies are relatively indifferent to different tracking systems, and generally deteriorate with the lengthening of the data arc. Furthermore, intuitive understanding of the orbit determination process accumulated through years of experience with noise-dominated measurements may no longer be applicable now that error sensitivities with respect to unmodeled parameters are of primary concern. For instance, results of "noise-only" covariance analysis give no indication of the reality of orbit errors, and injudicious accumulation of more measurements may actually result in poorer orbits.

Introduction

SATELLITE technology has progressed to the stage that employing high-altitude satellites as operational mobile tracking stations will be a reality in the 1980's. The primary advantage of these high-altitude tracking satellites over ground tracking stations is the much wider line-of-sight tracking and communication coverage they provide for low-altitude spacecraft (Fig. 1). The NASA Tracking and Data Relay Satellite System (TDRSS)[1] being built consists of two operational satellites in geosynchronous orbits spaced approximately 130-deg apart, at 41° and 171° W longitude. A TDRSS ground terminal located near White Sands, N. Mex., will operate with the satellites to provide a telecommunications service for transferring tracking, telemetry, command, voice, and image data between the ground and low-altitude user spacecraft. Basic tracking measurements provided are round-trip range and range-rate from the ground terminal to a geosynchronous relay satellite and then to the low-altitude user spacecraft. In contrast to the two-satellite geosynchronous TDRSS, a 63-deg inclination, 12-h-period multiple satellite global positioning system (GPS)[2] will be implemented by DOD. In the first phase, GPS will have six satellites distributed in two orbital planes. Eventually the total number will be increased to 24. GPS is meant to provide positioning information to users ranging from missiles, spacecraft, aircraft, and ship to tanks and foot soldiers. Each GPS satellite continuously radiates information about its ephemeris and time. A user receives the one-way signal, decodes the information, and computes its range to the GPS satellite by contrasting the time of signal transmission and reception. Thus the basic measurement in GPS is the one-way range from a user to the visible GPS satellites.

The orbital geometry assumed for the TDRSS and GPS Phase I spacecraft is shown in Tables 1 and 2. The modeling of the error sources is discussed in the next section, followed by extensive results on orbit determination error analyses and parametric studies.

Error Model

Orbits are computed from tracking measurements based on our mathematical models of the measurement process and orbital dynamics. Thus, orbital errors result if the measurements and the modeling are imperfect. Good orbit determination algorithms tend to desensitize the effect of these imperfections.

Measurements contain errors in the presence of instrument noise, calibration bias, propagation uncertainty, etc. Instrument noise level is already low with present systems such as the NASA Unified S-Band System, and will be reduced further in the 1980's. It will only take a small amount of redundant measurements to smooth out random errors. Systematic tracking errors such as calibration biases cannot be removed by simple smoothing and are more difficult to deal with. Sometimes, if sufficient information content exists in tracking measurements, the biases may be estimated along with the orbits. Frequently, however, one just has to live with some of the systematic errors. Modeling errors are even more troublesome because they may mean inaccuracies in the functional forms of the mathematical model. The measurement process model relates the measured quantities to the instantaneous orbit, and is relatively simple for range and range-rate measurements. The dynamic model relates satellite orbital states at different times, and therefore relates, indirectly, measurements made at different times. As the time separation gets longer, this relation becomes less accurate, and the orbital error resulting from dynamic modeling error becomes more serious. To be mathematically tractable, the

Table 1 Phase I GPS constellation

Satellite no.	Inclination, deg	Period, h	Node, deg	Mean anomaly, deg
1			−120	−23.2
2			−120	16.8
3	63	12	−120	56.8
4			120	16.8
5			120	56.8
6			120	96.8

Table 2 TDRS orbital geometry

Satellite	Period, h	Longitude, deg	Inclination
TDRS-E		319	
TDRS-W	24	189	within ±7 deg
TDRS-SPARE		260	

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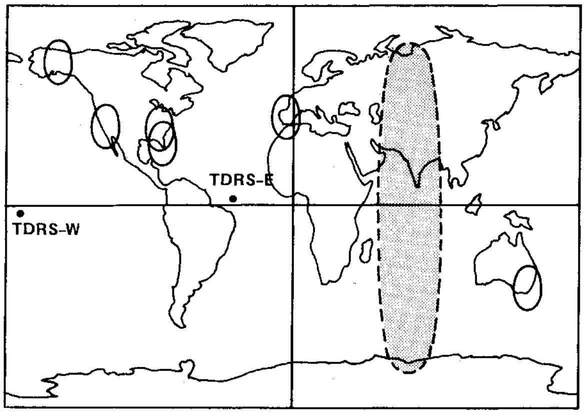

(TDRSS HAS GLOBAL COVERAGE OUTSIDE OF SHADED REGION. GROUND STATIONS COVER THE SMALL ELLIPTICAL REGIONS.)

Fig. 1 1980 NASA space flight tracking and data network orbital coverage at 200-km altitude.

modeling errors considered in this study are represented by a set of parameters which are not known with certainty. Table 3 presents our model of error sources and their standard deviations. This model is on the conservative side for the 1980's.

Certain clarifications of Table 3 are in order. The measurement noise level depends on the data sample smoothing time. The values given are consistent with TDRSS specifications.[1] Results to be presented later indicate orbital errors caused by the assumed noise level amount to only about 1% of the total error. Thus, the real significance of the assigned noise levels lies not in its direct error contribution, but in that their relative values specify the weights to be given to different tracking data; e.g., range vs range-rate in the orbit determination algorithm. The gravity model error in Table 3 refers to uncertainties in the spherical-harmonic coefficients of the nonhomogeneous, nonspherical Earth. It is assumed their standard deviations amount to 50% of the differences between the GEM-5[3] model and another model of comparable accuracy, but derived from a different, and thus uncorrelated, data set.

A source of error particular to satellite-to-satellite tracking is the tracking satellite ephemeris error, which varies with time. One may be tempted to represent these time-varying errors by some properly chosen errors at epoch, and consider them as a subset of the systematic error parameters discussed earlier. However, care must be exercised in the choice because improper ephocal errors may propagate to become unrealistic errors later on. In addition, there is a more fundamental

Table 3 Error model

Tracking measurements	Bias	Noise
Range, m	5	1
Doppler, mmls	0.1	0.15
	Station Location, m	
Latitude 5	Longitude 5	Height 5
Tropospheric Refraction	10%	(ground stations to TDRS only)
	Dynamic	
Atmospheric drag	25%	
Solar radiation pressure	15%	
Earth GM	1 part per million	
Gravity	50% of the difference between GEM-5 and another independently derived gravity model of comparable accuracy.	

difficulty to this approach. That is, many other error sources, such as the uncertainty in our knowledge of the geopotential, affect both the tracking and tracked satellites. Therefore, it is incorrect to consider the tracking satellite ephemeris error and the geopotential uncertainty as independent error sources in the orbit determinations of user (tracked) spacecraft.

The mathematics for the proper modeling of tracking satellite ephemeris error is given in Fang and Gibbs,[4] and represents additional complexity. Fortunately, for most practical situations, the effect may be simulated on existing multisatellite error analysis programs as follows. Set up an error analysis for a simultaneous orbit determination of tracking and tracked satellite orbits, with the ground tracking of tracking satellites given much greater weight over satellite-to-satellite tracking. Intuitively one should expect the tracking satellite orbits thus obtained to be independent of the satellite-to-satellite measurements, and yet the tracking satellite orbit errors will propagate into the tracked satellite orbit solution. This is indeed true as is proven rigorously in Ref. 4. This artificial weighting used will upset the contribution of random measurement noise. However, as discussed before, the noise contribution is insignificant anyway for most practical situations.

For the reasons just noted, the tracking satellite ephemeris errors are not given explicitly in Table 3. Instead these errors are considered to be those resulting from station coordinate error, range bias, solar pressure, and GM uncertainties, etc. Numerical values of the ephemeris errors to be expected are discussed in a later section.

As mentioned earlier, orbit determination errors also depend on the algorithm used for orbit computations. A discussion of orbit determination algorithms and their effect on orbital errors may be found in Fang.[5] Most proven orbit determination programs in existence are based on batch-processing, weighted least-squares estimation algorithms. Symbolically, one may write

$$x(t_0) = f(z, \alpha, e) \tag{1}$$

where $x(t_0)$ = orbital elements of epoch t_0, z = measurements, α = uncertain parameters, e = random measurement noises, f = orbit computation algorithm

If it is assumed the sources of errors are small and that the algorithm in Eq. (1) gives the correct orbital elements in the absence of errors, one obtains the following linearized equation relating the errors in the computed orbital elements at epoch and the error sources,

$$\Delta x(t_0) = \frac{\partial f}{\partial \alpha}\Delta\alpha + \frac{\partial f}{\partial e}\Delta e \tag{2}$$

Orbit errors at other times t may then be evaluated as

$$\Delta x(t) = \frac{\partial g}{\partial x(t_0)}\Delta x(t_0) + \frac{\partial g}{\partial \alpha}\Delta\alpha$$

$$= \left[\frac{\partial g}{\partial x(t_0)}\frac{\partial f}{\partial \alpha} + \frac{\partial g}{\partial \alpha}\right]\Delta\alpha + \left[\frac{\partial g}{\partial x(t_0)}\frac{\partial f}{\partial e}\right]\Delta e \tag{3}$$

if the orbital dynamics are assumed to be governed by the relation

$$x(t) = g[x(t_0), \alpha] \tag{4}$$

The functional forms f and g, although simple in appearance, actually stand for a rather involved iterative solution of nonlinear algebraic and differential equations. The coefficients of $\Delta\alpha$ and Δe in Eq. (3) represent orbit error sensitivities with respect to the uncertain parameter $\Delta\alpha$ and the random noise Δe, respectively. A large and versatile multiarc, multisatellite trajectory error analysis computer program

ORAN[6] is used to compute these error sensitivity coefficients. The contribution of each error source is identified separately and the root sum square (rss) of individual errors (standard deviations of errors given in Table 3 scaled by their error sensitivity coefficients) gives the overall orbital error.

It should be emphasized that orbit determination is a nonlinear estimation problem, which is usually solved by an iterative process. A linear error analysis such as that used in the present study presumes a converged orbit solution and that errors concerned are not sufficiently large to cause divergence. This is generally valid for precision orbit determination, which we are concerned with, and realistic orbital errors should result if the error model is realistic. Within the realm of validity of linearity, the results may be scaled up and down for other expected levels of the error sources.

Results and Discussions

Based on the error analysis method discussed in the preceding section, investigations were made to evaluate the ephemeris accuracies of the TDRSS and GPS tracking satellites and the performances of these systems in the determination of low-altitude user spacecraft orbits. A batch-weighted least-squares orbit determination process based on a one-day tracking arc is assumed, unless specifically stated otherwise. Major results will be summarized in the following sections. Additional details may be found in a series of reports by the author.[4,5,7,8]

Ephemeris Accuracy of High-Altitude Tracking Satellites

The primary error sources affecting the ephemerides of high-altitude tracking satellites are uncertainties in our knowledge of GM, solar radiation, tracking station coordinates, and range biases. The maximum rss position errors of TDRS orbits are approximately 100 m. For GPS satellites, they are around 30-40 m. The reasons for the better accuracy in GPS ephemeris are as follows.

1) GPS orbits are primarily range-rate solutions, while TDRS orbits, being geostationary, are range solutions. Range-rate solutions are much less sensitive to the GM error and, of course, are independent of range biases.

2) Station coordinate uncertainties tend to amplify in the ratio of "orbital radius/Earth radius," and have greater effect on the high-altitude TDRS satellites.

For this study it is assumed that bilateration is used for TDRS satellites, while GPS satellites are tracked, when visible, from four stations located at Guam, Hawaii, Elmendorf Air Force Base, Alaska, and Vandenberg Air Force Base, Calif. (A hybrid batch and sequential orbit determination process[2] is planned for the GPS satellites. For comparison with TDRSS, a batch process is assumed here.) Although the tracking frequency is unimportant, the geometry of tracking station baseline is. When the geometry is unfavorable, the ephemeris becomes sensitive to solar radiation errors. This results not only in larger orbit errors, but also in much greater fluctuations during the data arc. What constitutes favorable geometry is not always intuitively obvious. For bilateration of TDRS, the inclination and arc length of the great circle connecting the two tracking stations are important parameters. Good geometry generally means an arc length longer than 100 deg and an inclination less than 60 deg. Our results indicate good geometry is provided by the White Sands, N. Mex., and Honeysuckle, Australia, combination for TDRS-W and White Sands-Ascension Islands combination for TDRS-E. For the GPS satellites, the four tracking stations seem to provide a somewhat less favorable geometry for Satellite #1 at the assumed epoch. The one-day orbit prediction accuracies are approximately 300 m and 80 m for the TDRS and the GPS spacecraft, respectively.

Table 4 Comparison of TDRSS and GPS long-arc orbit determination accuracy for SEASAT (774 km, 82-deg inc)[a]

Tracking arc length, h	Tracking system	rss orbital uncertainty during data arc: Maximum pos, m	Maximum vel, cm/s	Average pos, m	Average vel, cm/s
6	TDRS	28	2.5	23	2.0
	GPS	22	1.9	15	1.3
12	TDRS	29	2.6	19	1.7
	GPS	23	2.1	16	1.3
24	TDRS	31	3.0	18	1.7
	GPS	32	2.6	14	1.3

[a]Based on range and range-rate measurements to visible satellites at 30-min intervals. Therefore there are more measurements by GPS than TDRS.

Low-Altitude Spacecraft Orbit Determination Based on Satellite-to-Satellite Tracking

Comparable Performance of TDRSS and GPS

It will be shown that orbit determination of low-altitude satellites based on a long arc (longer than a few low satellite orbits) satellite-to-satellite tracking is limited in its accuracy not by tracking errors but by dynamic modeling errors. Thus, the orbit determination performances of TDRSS and GPS are comparable (see Table 4), even though the GPS spacecraft ephemerides are more accurate than the TDRS ephemerides. Exceptions to this statement exist only for short-arc tracking, and for tracking of equatorial user spacecraft, when tracking geometry provided by TDRSS is unfavorable.

Satellite-to-Satellite Tracking Orbit Determination Performance and Error Sources

Error analyses were conducted for various classes of low-Earth-orbit spacecraft planned for the 1980's. The breakdown of error sources for two representative spacecraft is shown in Table 5 together with the tracking TDRS ephemeris error breakdown. Details for other orbital geometries may be found in Ref. 4. These results are summarized in the following:

1) Random measurement noise is negligible in comparison with systematic errors. Even at a low data rate of one set of measurements every 2 h, for a total of 24 measurements, the 1-m range and 0.15 mm/s range-rate measurement noises contribute to less than 1% of the total orbit error.

2) Among the systematic errors, uncertainties in the dynamic model are predominant.

3) Drag uncertainty dominates orbits under 550-km altitude. If drag is not estimated it is difficult to achieve: a) 10-km position accuracy for orbits under 300-km altitude, b) 1-km position accuracy for orbits under 400-km altitude. However, there is enough information content, even at the 24-measurement low data rate, to solve for an effective drag coefficient. Once drag is solved for,† the remaining error is primarily the result of the uncertainty in gravity harmonics. For low inclination orbits at lower than 300-km altitude, the gravity harmonics uncertainty may cause errors exceeding 1 km.

4) At altitudes from 550 to 1000 km, the uncertainty in gravity harmonic coefficients becomes dominant. Position errors decrease from approximately 120 m to 25 m with altitude. Generally the effect of gravity harmonics uncertainty tends to increase with the decrease of the orbital inclination.

5) The orbit error for a spacecraft at 400-km altitude and 0-deg inclination is considerably greater than another spacecraft at the same altitude, but at 30-deg inclination. This is a result of the poor geometry, since the equatorial orbit is nearly coplanar with the TDRS orbits assumed at ±2-deg inclination. By locating the TDRS orbits at ±5-deg in-

†Of course any drag uncertainty not accounted for by an effective drag coefficient will remain.

Table 5 Maximum orbital position error and its composition[a]

Error source and level	SAT-A 980 km, 99 deg inc	SAT-B 300 km, 20 deg inc, Drag estimated	TDRS-E (319°E long)	TDRS-W (189°E long)
	Position uncertainty, rounded to the nearest meter			
Dynamic				
GM (1 part per million)	7	15	66	68
Geopotential coeff. (50% of MD 1 and GM 5 diff.)	23	702	0	0
Solar Radiation (15%)				
TDRS-E	3	3	60	...
TDRS-W	3	2	...	29
Drag (25%)	1	...	0	0
Tracking				
White Sands				
Longitude (5 m)	3	3	8	32
Latitude (5 m)	1	1	2	8
Height (5m)	1	0	2	7
Range bias (5 m)	3	2	8	34
Range-rate bias (0.1 mm/s)	0	0	0	0
Trop. refraction (10%)	1	1	2	8
Johannesburg[b]				
Longitude (5 m)	2	3	30	...
Latitude (5 m)	0	0	5	...
Height (5 m)	0	0	6	...
Range bias (5 m)	2	3	31	...
Range-rate bias (0.1 mm/s)	0	0	0	...
Trop. refraction (10%)	0	1	8	...
Honeysuckle				
Longitude (5 m)	1	0	...	7
Latitude (5 m)	1	0	...	5
Height (5 m)	1	0	...	6
Range bias (5 m)	2	1	...	11
Range-rate bias (0.1 mm/s)	0	0	...	0
Trop. refraction (10%)	0	0	...	1
Relay				
Range bias (5 m)	0	0	...	...
Range-rate bias (0.1 mm/s)	0	0	...	...
Random Noise				
(1 m and 0.15 mm/s standard deviation in range and range-rate measurements)	0	1	1	1
Total error (rss)	25	702	100	90

[a]Orbital errors shown are those at the time of maximum rss error, and may not represent the maxima for individual error sources. The effects of TDRS ephemeris errors on SAT-A and B are represented indirectly by solar radiation and tracking station errors, etc. The GM uncertainty contains both the direct and indirect effects, although the former is greater.
[b]Because of political considerations, Johannesburg is not considered a candidate tracking station anymore. Replacement of Johannesburg by Ascension Island results in a slightly inferior performance.

Table 6 Orbital accuracy for different tracking configurations[a]

Tracking configuration	SAT-A 99-deg inc, 980 km Total error	SAT-A Leading error sources	SAT-B 20-deg inc, 300 km (drag estimated) Total error	SAT-B Leading error sources
	Maximum position uncertainty, m			
TDRS-E&W	25	23 (Grav. H) 10 (GM)	702	702 (Grav. H) 15 (GM)
TDRS-W only	37	31 (Grav. H) 23 (GM)	583	583 (Grav. H) 19 (GM)
TDRS-E and SPARE	25	22 (Grav. H) 8 (GM)	852	852 (Grav. H) 16 (GM)
TDRS-W and SPARE	33	23 (Grav. H) 21 (GM)	561	561 (Grav. H) 17 (Solar pressure, TDRS-SPARE)

[a] A one-day tracking at a rate of 10-min every 2 h.

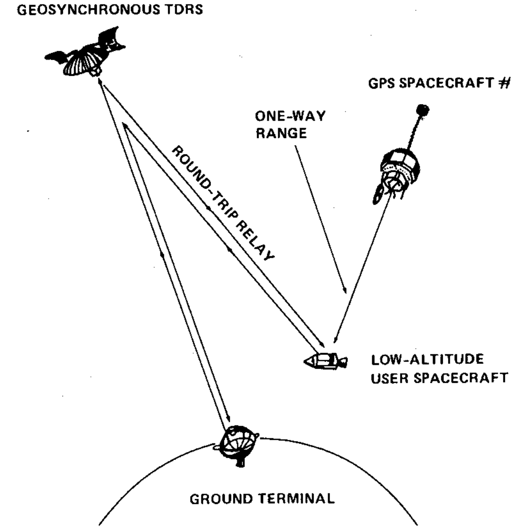

Fig. 2 Satellite-to-satellite tracking measurements.

Table 7 SEASAT accuracy vs tracking arc length[a]

Arc length, min	Position error, m Phase I GPS		TDRSS	
60	12.34	9.26	...	...
105	8.65	7.24	18.3	13.3
240	22.21	11.03	16.60	11.80
360	...	...	15.91	12.54
480	...	...	20.20	14.30

[a]Results in this table are based on range-rate tracking. Combined range and range-rate tracking results agree with those values within 10%. Tracking at 15-min intervals. Tracking at 5-min intervals does not give better results.

clinations, the orbit error is halved. This brings out the difficulty of obtaining highly accurate equatorial orbits from TDRSS.

6) Among the tracking errors, the TDRS orbit errors as a result of range bias and station longitude uncertainty contribute the most to the user orbit error. They tend to increase somewhat with user altitude. Range biases and other tracking errors in the relay measurements are not as significant.

Tracking Mode Parametric Study Results

Tracking Geometry

Good tracking geometry is important. With high-altitude tracking spacecraft, the advantages of long-baseline navigation are realized. For long data arcs (a few low satellite orbital periods), the high-altitude spacecraft orbital radius is the effective baseline length. Thus, the effect of the high-altitude tracking spacecraft position error on that of the low-altitude tracked satellite is scaled down by a ratio of their respective orbital radii. This leads to the important conclusion that moderate degradation of tracking satellite ephemeris does not adversely affect user spacecraft accuracy. Furthermore, being a *mobile* station, over a long data span, a single tracking spacecraft provides sufficient geometry for the tracking of low-altitude satellites, if visibility is not a problem, as it is generally true for the geosynchronous TDRS spacecraft. This is a comforting thought in the event of an unexpected failure of one of the TDRS's. A comparison of single-TDRS and two-TDRS performance is given in Table 6.

The six-satellite GPS provides inherently good geometry. The two-geosynchronous satellite TDRSS has unfavorable geometry for equatorial user satellites and for tracking arcs shorter than one-half of user spacecraft period.

Optimum Tracking-Arc Length

Orbit uncertainties resulting from measurement errors are "geometric" in nature and will benefit from a longer tracking arc. On the other hand, dynamic modeling errors tend to amplify with time, and therefore, with the length of the tracking arc. Optimum tracking lengths are compromises between these two conflicting requirements. The optimum

Table 8 Effect of tracking rate on orbital accuracy

	SAT-A, 99-deg inc, 980 km		SAT-B (drag estimated) 20-deg inc, 300 km		
	Position uncertainty, m				a posteriori uncertainty in drag, %
	Total error	Leading error sources	Total error	Leading error sources	
Continuous (3108 R/Rt meas. for SAT-B, 4226 for SAT-A)[b]	18	16 (Grav. H)[a] 10 (GM)	465	465 (Grav. H) 14 (GM)	0.6
10 min/h (426 R/Rt meas. for SAT-B, 736 for SAT-A)	20	19 (Grav. H) 10 (GM)	568	568 (Grav. H) 15 (GM)	1.0
10 min/2 h (238 R/Rt meas. for SAT-B, 80 for SAT-A)	25	23 (Grav. H) 10 (GM)	702	702 (Grav. H) 15 (GM)	1.3
2 min/2 h (60 R/Rt meas. for SAT-B, 80 for SAT-A)	25	23 (Grav. H) 10 (GM)	633	623 (Grav. H) 22 (Solar rad TDRS-E)	1.0
2 min/4 h (40 R/Rt meas. for SAT-B, 48 for SAT-A)	25	22 (Grav. H) 13 (GM)	566	566 (Grav. H) 35 (Solar rad TDRS-E)	0.9
2 min/6 h (24 R/Rt meas. for SATS-H, 32 for EOS-B)	30	26 (Grav. H) 16 (Solar rad TDRS-E)	2351	2349 (Grav. H) 76 (Solar rad TDRS-E)	0.3
Once/2 h (24 R/Rt meas.)	30	28 (Grav, H) 12 (GM)	1143	1143 (Grav. H) 19 (GM)	1.3
10 R/Rt meas. (approx. once/6 h)	not computed		1536	1536 (Grav. H) 28 (Solar rad TDRS-E)	2
8 R/Rt meas. (approx. once/6 h)	not computed		1103	1095 (Grav. H) 195 (Solar rad TDRS-E)	0.8

[a]Grav. H stands for geopotential harmonic coefficients. [b]R/Rt stands for range and range rate.

tracking arc lengths for a user spacecraft with orbital geometry similar to that of SEASAT are given in Table 7. It is seen that for the TDRSS, about two SEASAT orbital periods are all that are requried for geometric observability. Further increases in tracking arc lengths result in greater errors from dynamic model uncertainties. For the Phase I GPS, the six tracking satellites provide better geometry, and one SEASAT orbital period of tracking gives the best accuracy. These results should be representative of above-the-atmosphere, low-altitude, high-inclination, circular orbit users. If user ephemeris is used for orbit *prediction* purposes, a longer tracking arc would be beneficial.

Data Rate

The effect of data rate on orbital accuracies of two representative user spacecraft is presented in Table 8. The results amply demonstrate the fact that when measurement noise level is very low, measurement density is really of no consequence. *It is much more important to have measurement distribution which provides good geometry and observability.* This is the distinct advantage provided by satellite-to-satellite tracking. Also, different measurements may have different error sensitivities with respect to unmodeled error sources. By weighing measurements indiscriminant of their error sensitivities, the use of more tracking data may even hurt the orbital accuracy. The SATELLITE-B orbit, based on eight well-distributed measurements, demonstrates the situation dramatically. Since six orbital elements plus the drag coefficient are solved for, this orbit is essentially a deterministic solution. Yet its accuracy is better than several other orbits based on more frequent measurements, and is not far behind that based on 3108 continuous measurements.

Range vs Range-Rate Tracking

As stated before, a batch "minimum variance" weighted least-squares orbit determination process is simulated in our error analysis. When unmodeled errors rather than random measurement noises are predominant, weighing the measurements according to the measurement noise variance is generally nonoptimum. There is sufficient information for the determination of a user orbit based on range measurements alone, or range-rate measurements alone. The user orbit based on combined range and range-rate tracking may be viewed as an orbit obtained by appropriately weighing the range-only and range-rate-only orbits. Table 9 presents a comparison of errors in the SATELLITE A and B orbits based on range tracking, range-rate tracking and combined range and range-rate tracking, respectively. By examining the noise only σ of the orbital error one sees that, with the assumed 1-m range and 0.15-mm/s range-rate measurement standard deviations in our error model, the combined orbit is more of a range-rate solution than a range solution. For SAT-A, the range solution is much more sensitive to errors than the range-rate solution. For SAT-B both solutions are somewhat comparable, with the range solution slightly better. Most likely the difference in their behaviors is a result of the difference in the orbital inclinations. The results also point out the care one should exercise in weighing different data in the low-noise-measurement 1980 era. For instance, had the range noise level been ten times smaller, it would still be better off for the SAT-A orbit to assign relative weights according to the present scheme rather than the "minimum variance" way. In general, range-rate orbits are less sensitive to the GM error and to unfavorable tracking geometry.

GPS User Spacecraft Clock Offset Determination

User spacecraft range to the GPS spacecraft is based on the one-way radio frequency signal propagation time. A 10-ns timing error is equivalent to a 3-m range bias. Thus, the user spacecraft clock should be determined to high accuracy. The possibility of determining user clock offset as a part of the orbit solution is investigated. In the absence of unmodeled dynamic parameter errors, clock biases may be determined to the nanosecond level. There is, however, a high correlation between the clock bias and the GM uncertainty. Thus, it is not possible to determine clock bias to better than 25-ns accuracy in the presence of a gravitational error of one part per million. This result illustrates some of the difficulties of attempting to estimate uncertain parameters along with the orbital elements. Many of the parameters manifest their effect on the satellite orbital period and may not be easily distinguished.

Simultaneous Solution for User and TDRS Orbits

In operational orbit determination, it is not anticipated that the relay tracking of user and the ground tracking of TDRS will be processed together to solve for both TDRS and user orbits simultaneously. Still, there have been suggestions that perhaps a simultaneous solution may improve the orbits or

Table 9 Comparison of orbits based on range and rate tracking, range tracking, and range-rate tracking

			SAT-A 99-deg inc, 980 km	SAT-B, 20-deg inc, 300 km[a]
Range and range-rate tracking	Position uncertainty, m	Overall	25	702
		contribution due to noise	0.06	0.12
	Dominant error sources		23 (Grav. H)[b] 10 (GM)	702 (Grav. H) 15 (GM)
Range tracking	Position uncertainty, m	Overall	76	633
		contribution due to noise	0.59	0.72
	Dominant error sources		67 (Solar rad TDRS-E) 23 (Grav. H) 21 (White Sands Range Bias)	633 (Grav. H) 53 (Solar rad TDRS-E)
Range-rate tracking	Position uncertainty, m	Overall	26	758
		contribution due to noise	0.06	0.15
	Dominant error sources		24 (Grav. H) 10 (GM)	758 (Grav. H) 16 (GM)

[a] Drag is estimated for SAT-B. [b] Grav. H stands for gravity harmonic coefficients.

Table 10 Orbit errors in simultaneous orbit solutions[a]

	Maximum position uncertainty, m			
	Simultaneous solutions			Separate solutions
	Continuous relay tracking	10 min/h relay tracking	10 min/2 h relay tracking	10 min/2 h relay tracking
SAT-A (980 km, 99-deg inc)	45	35	33	32
TDRS-E	240	106	90	110
TDRS-W	221	99	78	96
SAT-C (610 km, 29-deg inc)	108	89	81	72
TDRS-E	637	401	299	110
TDRS-W	581	326	433	96

[a] Ground tracking of TDRS at 5 min/2 h rate.
[b] The error model used, and therefore the orbital errors shown in this table, are slightly different from those presented in other tables.

reduce the tracking requirements. To clarify this point, error analyses were made and the results are presented in Table 10 for two different user spacecraft. For comparison purposes, corresponding results for a separate user solution are given in the last column of the table. It is interesting to note, from this table, that 1) simultaneous solutions do not seem to give better orbital accuracy, and 2) in the simultaneous solutions, more frequent relay tracking results in poorer orbits. This seemingly paradoxical result comes about because the relay measurements also contain information about TDRS orbits, and yet this information is highly sensitive to modeling errors. More frequent relay measurements only serve to give more weight to these error-sensitive measurements.

Conclusions

Operational orbit determination based on satellite-to-satellite tracking will be a reality in the 1980's. Extensive error analyses and parametric studies were undertaken to clarify the characteristics, interactions, and errors in the orbit determination of the high-altitude tracking and low-attitude tracked spacecraft. The most important finding of the study may be stated as follows: The major sources of orbit determination errors are uncertainties in dynamic model parameters. Systematic tracking errors are secondary and random measurement noises are significant. Unfavorable tracking geometry also tends to accentuate the effect of dynamic modeling error. Yet, because of correlations of errors or lack of sufficient information content, generally there is not enough observability to estimate all the uncertain parameters of consequence.

This finding has important consequences on many facets of the orbit determination process:

1) Tracking rates much reduced from those customarily employed at present are permissible. This would lessen the operational activities of tracking, require less data communication and storage, and also reduce orbit computation times.

2) As long as tracking geometry is favorable, orbit determination accuracies are relatively indifferent to different tracking systems. Thus, the two planned tracking systems, TDRSS and GPS, have comparable performances in long-arc (a few tracked satellite orbital periods) orbit determination. As a matter of fact, tracking by just a single geosynchronous TDRS satellite generally provides sufficient geometry for a good orbit resolution. This means that even a catastrophic failure of one of TDRSS satellites will not cripple the whole system. In short-arc tracking and for tracking equatorial orbits, the inherently better geometry provided by the multisatellite GPS results in more accurate orbits. In general, because of the good line-of-sight coverage by the high-altitude tracking satellites, satellite-to-satellite tracking geometry compares favorably with a ground network with a small number of tracking stations.

3) Since dynamic modeling error tends to amplify with time, orbit accuracy generally deteriorates with a lengthening of the tracking arc. If orbital accuracy is the primary consideration, the tracking arc should only be as long as that required by geometry and observability, usually no more than one or two orbital periods.

4) Intuitive understanding of the orbit determination process accumulated through years of experience with noise-dominated measurements may no longer be applicable now that error sensitivities with respect to unmodeled parameters are of primary concern. For instance: results of "noise-only" covariance analysis give no indication of the reality of orbit errors; injudicious accumulation of more measurements may actually result in poorer orbits; the specification of measurement noise levels affects orbit determination not in the direct noise contribution to errors but in the relative weights it assigns to different sets of measurements. This also means that if orbital accuracy is of primary concern, error analyses would have to be conducted for *individual satellites* in order to determine the best tracking strategy.

Another finding of some interest is the role played by the (orbital radius/tracking station radius) ratio. Station coordinate errors are mapped into satellite orbit errors, scaled by this ratio. For instance, in the ground tracking of the geosynchronous TDRS satellites, a 5-m station coordinate error becomes approximately $6\times$ (geosynchronous radius/Earth radius) ≈ 30 m. On the other hand, in satellite-to-satellite tracking, the geosynchronous TDRS's play the role of mobile tracking stations, and the effect of TDRS ephemeris error is scaled down by the ratio (low-altitude satellite radius/geosynchronous radius) $\approx 1/6$. This is the reason TDRS ephemeris uncertainties are secondary sources of error and a moderate degradation of TDRS ephemeris does not adversely affect tracked satellite accuracy.

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